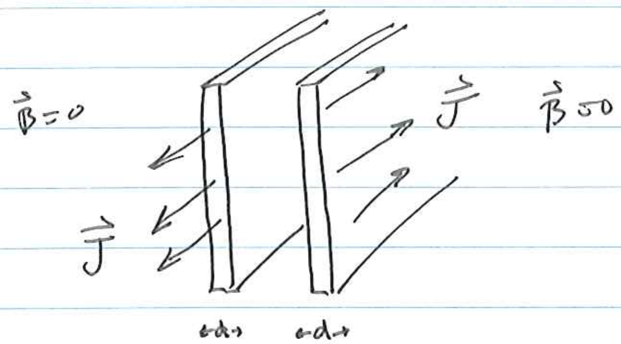
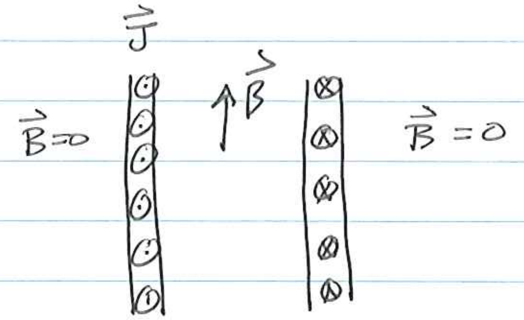
Review: Final exam

Problem 1: Consider two parallel plates with currents flowing in opposite directions. What is B?
Each plate has current density $\vec{J}$ and thickness d.



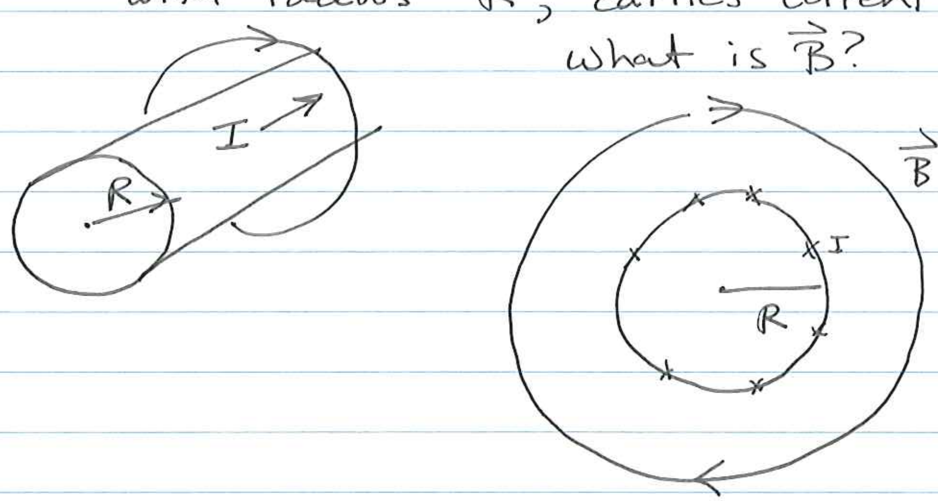


Using Ampere's law: for each plate

$$B = \frac{\mu_0 I_{encl}}{L} = \frac{\mu_0 J l d}{2l} = \frac{\mu_0 J d}{2}$$

$$\Rightarrow \text{total } B = \mu_0 J d$$

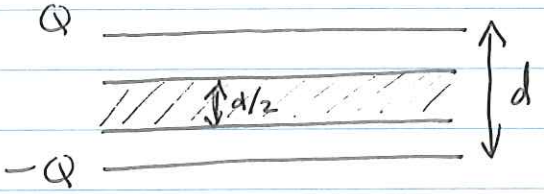
Problem 2: Thin conducting cylindrical shell with radius R, carries current I. What is $\vec{B}$?



Problem 3: Consider a parallel plate capacitor with area A and separation distance d.

What is C? $\Delta\phi = Ed = \frac{Q}{A\varepsilon_0} d \rightarrow C = \frac{A\varepsilon_0}{d}$

Suppose we insert a neutral conducting slab with thickness $d/2$ and area A between the plates.

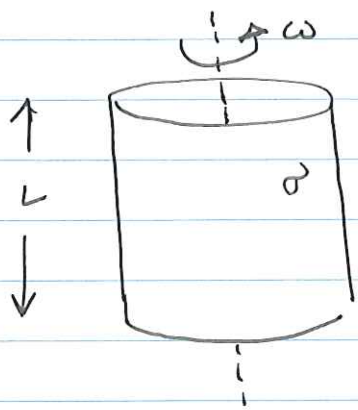


What are the surface charges on the slab? $\pm Q/A$.

What is the new capacitance? $d \rightarrow d/2$

$$\Rightarrow C = \frac{2A\varepsilon_0}{d}$$

Problem 4: ~~[illegible]~~ Thin cylindrical shell with surface charge σ. If shell is rotating with angular frequency ω, what is $\vec{B}$?



Like a solenoid: $\vec{B} = nI\mu_0\hat{z}$

$$nI = \frac{\text{number of windings}}{\text{length}} \times \frac{\text{current}}{\text{winding}}$$

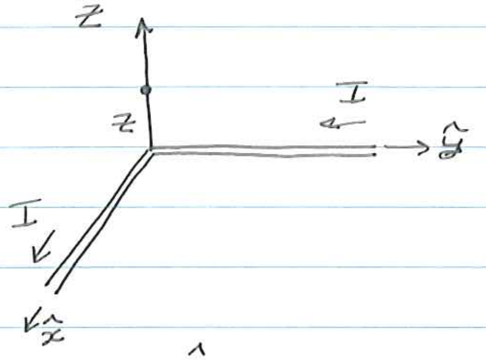
$$= \frac{\text{total current ~~around~~ over whole length}}{\text{length}}$$

$$= \frac{\text{total charge}}{\text{length} \cdot \text{time}} = \frac{\text{charge}}{\text{length time circum.}} \frac{\text{circumference}}{}$$

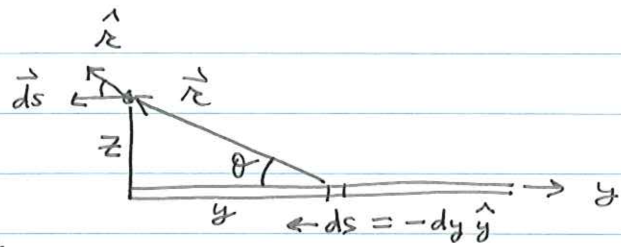
$$= \frac{\sigma\, 2\pi RL}{L\, 2\pi/\omega} = \frac{\sigma R \omega L}{L} = \sigma R\omega$$

$$\boxed{\vec{B} = \mu_0 \sigma R\omega\, \hat{z}}$$

Problem 5: Bent wire. What is $\vec{B}$ at height z above origin?



Consider just the segment along y-axis.
Use Biot-Savart law for half-infinite wire.



note: $\vec{ds} \times \hat{r} = -\hat{x}\sin\theta\, dy$

y-segment

$$\vec{B} = \int \frac{\mu_0 I}{4\pi}\frac{\vec{ds}\times\hat{r}}{r^2} = \int_0^\infty -\frac{\mu_0 I}{4\pi r^2}\, dy \sin\theta$$

$$= -\int_0^\infty dy \frac{\mu_0 I}{4\pi}\frac{z\,\hat{x}}{(z^2+y^2)^{3/2}} = -\frac{1}{2}\underbrace{\int_{-\infty}^{\infty} dy \frac{\mu_0 I}{4\pi}\frac{z}{(z^2+y^2)^{3/2}}}_{\text{same as for infinite wire}}\hat{x}$$

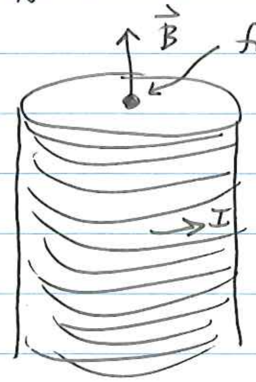
$$= -\frac{\mu_0 I}{4\pi z}\hat{x}$$

x-segment: $\vec{B} = -\frac{\mu_0 I}{4\pi z}\hat{y}$

total: $\vec{B} = -\frac{\mu_0 I}{4\pi z}(\hat{x}+\hat{y})$

Problem 6: Half-Infinite solenoid. What is $\vec{B}$?

find $\vec{B}$ at center point at entrance to solenoid.



$$\vec{B} = \frac{1}{2} n I \mu_0 \hat{z}.$$